Continuity and Differentiability

1. Let $f : D \to \mathbb{R}$ be a function, where D is an open interval in $\mathbb{R}$ and let $c \in D$.

(i) f is said to be **continuous at $x = c$** if $\lim\limits_{x\to c+} [f(x)] = \lim\limits_{x\to c-} f(x) = f(c)$.

(ii) f is said to be **right continuous at $x = c$** (or **continuous from right at $x = c$**) if $\lim\limits_{x\to c+} [f(x)] = f(c)$.

(iii) f is said to be **left continuous at $x = c$** (or **continuous from left at $x = c$**) if

$$\lim_{x\to c-} [f(x)] = f(c).$$

2. f is said to be **discontinuous** at $x = c$ if it is not continuous at $x = c$.

3. Let $f : D \to \mathbb{R}$ be a function, where $D \subseteq \mathbb{R}$. Then,

(i) f is said to be **continuous on an open interval (a, b)** if f is continuous at every point of the interval (a, b), i.e., if for every $c \in (a, b)$, we have $\lim\limits_{x\to c+} [f(x)] = \lim\limits_{x\to c-} [f(x)] = f(c)$.

(ii) f is said to be **continuous on a closed interval $[a, b]$** if f is continuous at every point of the interval $[a, b]$, i.e., if

(a) f is continuous at every point of the open interval (a, b).

(b) f is right continuous at $x = a$, i.e., $\lim\limits_{x\to a+} [f(x)] = f(a)$.

(c) f is left continuous at $x = b$, i.e., $\lim\limits_{x\to b-} [f(x)] = f(b)$.

(iii) f is said to be **continuous everywhere** if f is continuous at every point of the real line.

4. Let $f : D \to \mathbb{R}$ be a function, where D is an open interval in $\mathbb{R}$ and let $c \in D$. Then,

(i) f is said to be **differentiable at $x = c$** if both $\lim\limits_{h\to 0} \dfrac{f(c+h) - f(c)}{h}$ and $\lim\limits_{h\to 0} \dfrac{f(c-h) - f(c)}{-h}$ exist finitely and are equal.

This limit is called *derivative* of f at $x = c$ and is denoted by $f'(c)$.

(ii) f is said to be **right differentiable at $x = c$** (or **differentiable from right at $x = c$**) if $\lim\limits_{h\to 0} \dfrac{f(c+h) - f(c)}{h}$ exists finitely.

This limit is called *right-hand derivative* of f at $x = c$ and is denoted by $Rf'(c)$.

(iii) f is said to be **left differentiable at $x = c$** (or **differentiable from left at $x = c$**) if $\lim\limits_{h\to 0} \dfrac{f(c-h) - f(c)}{-h}$ exists finitely.

This limit is called *left-hand derivative* of f at $x = c$ and is denoted by $Lf'(c)$.

5. The function f is differentiable at $x = c$ if it is both left differentiable and right differentiable at $x = c$, i.e., if $Lf'(c) = Rf'(c)$.

6. Let $f : D \to \mathbb{R}$ be a function, where $D \subseteq \mathbb{R}$. Then,

(i) f is said to be **differentiable on an open interval (a, b)** if f is differentiable at every point of the interval (a, b),

i.e., if for every $c \in (a, b)$, we have $Lf'(c) = Rf'(c)$.

(ii) f is said to be **differentiable on a closed interval $[a, b]$** if f is differentiable at every point of the interval $[a, b]$, i.e., if

(a) f is differentiable at every point of the open interval (a, b).

(b) f is right differentiable at $x = a$, i.e., $\lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$ exists finitely.

(c) f is left differentiable at $x = b$, i.e., $\lim\limits_{h\to 0} \dfrac{f(b-h) - f(b)}{-h}$ exists finitely.

(iii) f is said to be **differentiable everywhere** if f is differentiable at every point of the real line.

7. Sum, Difference, Product, Quotient and Composition of two continuous functions are continuous.

8. Sum, Difference, Product, Quotient and Composition of two differentiable functions are differentiable.

9. Every differentiable function is continuous, but a continuous function may not be differentiable.

Differentiation – I

1. Some important rules of differentiation are:

(i) $\frac{d}{dx}(N) = 0$, where N is a constant.

(ii) $\frac{d}{dx}\left(x^N\right) = Nx^{N-1}$, where N is a constant.

(iii) $\frac{d}{dx}(\log F) = \frac{\frac{d}{dx}(F)}{F}$, where F is a function of x.

(iv) $\frac{d}{dx}\left(e^F\right) = e^F \frac{d}{dx}(F)$, where F is a function of x.

(v) $\frac{d}{dx}\left(N^F\right) = N^F (\log N) \frac{d}{dx}(F)$, where N is a constant and F is a function of x.

(vi) $\frac{d}{dx}(\sin F) = \cos F \frac{d}{dx}(F)$.

$\frac{d}{dx}(\cos F) = -\sin F \frac{d}{dx}(F)$.

$\frac{d}{dx}(\tan F) = \sec^2 F \frac{d}{dx}(F)$.

$\frac{d}{dx}(\cot F) = -\operatorname{cosec}^2 F \frac{d}{dx}(F)$.

$\frac{d}{dx}(\sec F) = \sec F \tan F \frac{d}{dx}(F)$.

$\frac{d}{dx}(\operatorname{cosec} F) = -\operatorname{cosec} F \cot F \frac{d}{dx}(F)$.

(Here, F is a function of x and x is in radians.)

(vii) $\frac{d}{dx}\left(\sin^N F\right) = N \sin^{N-1} F \frac{d}{dx}(\sin F)$.

$\frac{d}{dx}\left(\cos^N F\right) = N \cos^{N-1} F \frac{d}{dx}(\cos F)$.

$\frac{d}{dx}\left(\tan^N F\right) = N \tan^{N-1} F \frac{d}{dx}(\tan F)$.

$\frac{d}{dx}\left(\cot^N F\right) = N \cot^{N-1} F \frac{d}{dx}(\cot F)$.

$\frac{d}{dx}\left(\sec^N F\right) = N \sec^{N-1} F \frac{d}{dx}(\sec F)$.

$\frac{d}{dx}\left(\operatorname{cosec}^N F\right) = N \operatorname{cosec}^{N-1} F \frac{d}{dx}(\operatorname{cosec} F)$.

(Here, N is a constant, F is a function of x and x is in radians.)

(viii) $\frac{d}{dx}(F_1 + F_2 - F_3) = \frac{d}{dx}(F_1) + \frac{d}{dx}(F_2) - \frac{d}{dx}(F_3)$. **(Simple Rule)**

(Here, F_1, F_2, F_3 are functions of x.)

(ix) $\frac{d}{dx}(NF) = N\frac{d}{dx}(F)$, where N is a constant and F is a function of x.

(x) $\frac{d}{dx}(F_1F_2) = F_1 \frac{d}{dx}(F_2) + F_2 \frac{d}{dx}(F_1)$. **(Product Rule)**

$\frac{d}{dx}(F_1F_2F_3) = F_1F_2 \frac{d}{dx}(F_3) + F_1F_3 \frac{d}{dx}(F_2) + F_2F_3 \frac{d}{dx}(F_1)$.

(Here, F_1, F_2, F_3 are functions of x.)

(xi) $\dfrac{d}{dx}\left(\dfrac{F_2}{F_1}\right) = \dfrac{(F_1)\dfrac{d}{dx}(F_2) - (F_2)\dfrac{d}{dx}(F_1)}{(F_1)^2}$. **(Quotient Rule)**

(Here, F_1, F_2 are functions of x.)

(xii) $\dfrac{d}{dx}(F^N) = NF^{N-1}\dfrac{d}{dx}(F)$, where N is a constant and F is a function of x.

(xiii) $\dfrac{d}{dx}\left(\sqrt{F}\right) = \dfrac{\dfrac{d}{dx}(F)}{2\sqrt{F}}$, where F is a function of x.

(xiv) $\dfrac{d}{dx}[\log(F_1F_2)] = \dfrac{d}{dx}[\log F_1 + \log F_2]$.

$\dfrac{d}{dx}\left[\log\left(\dfrac{F_1}{F_2}\right)\right] = \dfrac{d}{dx}[\log F_1 - \log F_2]$.

$\dfrac{d}{dx}\left[\log(F_1)^{F_2}\right] = \dfrac{d}{dx}[F_2 \log F_1]$.

$\dfrac{d}{dx}\left[\log_{F_2}(F_1)\right] = \dfrac{d}{dx}\left[\dfrac{\log F_1}{\log F_2}\right]$.

$\dfrac{d}{dx}\left[e^{\log F}\right] = \dfrac{d}{dx}(F)$.

(Here, F_1, F_2, F are functions of x.)

2. If x and y are connected by an expression of the form $f(x, y) = 0$, then we say that y is defined implicitly in terms of x. y is called *implicit function* of x.

3. If x and y are two functions in a single variable θ, say $y = f(\theta)$ and $x = g(\theta)$, then the functions x and y are called *parametric functions* and θ is called the *parameter*.

4. For a function in the parametric form, say $y = f(\theta)$, $x = g(\theta)$, we have

$$\frac{dy}{dx} = \frac{\left(\dfrac{dy}{d\theta}\right)}{\left(\dfrac{dx}{d\theta}\right)}, \text{ provided } \frac{dx}{d\theta} \neq 0.$$

5. We have the following formulae for the derivatives of inverse trigonometric functions in their respective domains:

(i) $\dfrac{d}{dx}(\sin^{-1}A) = \dfrac{1}{\sqrt{1-A^2}}\dfrac{d}{dx}(A)$

(ii) $\dfrac{d}{dx}(\cos^{-1}A) = -\dfrac{1}{\sqrt{1-A^2}}\dfrac{d}{dx}(A)$

(iii) $\dfrac{d}{dx}(\tan^{-1}A) = \dfrac{1}{(1+A^2)}\dfrac{d}{dx}(A)$

(iv) $\dfrac{d}{dx}(\cot^{-1}A) = -\dfrac{1}{(1+A^2)}\dfrac{d}{dx}(A)$

(v) $\dfrac{d}{dx}(\sec^{-1}A) = \dfrac{1}{A\sqrt{A^2-1}}\dfrac{d}{dx}(A)$

(vi) $\dfrac{d}{dx}(\text{cosec}^{-1}A) = -\dfrac{1}{A\sqrt{A^2-1}}\dfrac{d}{dx}(A)$

where A denotes any function of x.

6. If $z = f(u)$ and $u = g(x)$, then $\dfrac{dz}{dx} = \dfrac{dz}{du}\cdot\dfrac{du}{dx}$. This rule is called *Chain Rule* for finding the derivative of composition of two functions.

7. If $z = f(u)$, $u = g(v)$ and $v = h(x)$, then $\dfrac{dz}{dx} = \dfrac{dz}{du}\cdot\dfrac{du}{dv}\cdot\dfrac{dv}{dx}$. This rule is called *Chain Rule* for finding the derivative of composition of three functions.

8. If $y = f(x)$ and $z = g(x)$, then derivative of $f(x)$ w.r.t. $g(x)$ is given by

$$\frac{dy}{dz} = \frac{\left(\dfrac{dy}{dx}\right)}{\left(\dfrac{dz}{dx}\right)}, \text{ provided } \frac{dz}{dx} \neq 0.$$

Differentiation – II

1. If a function $f(x)$ is differentiable, then its derivative $f'(x)$ is known as the first order derivative of f. If the function $f'(x)$ is again differentiable, then its derivative $f''(x)$ is known as the second order derivative of f.

2. If x and y are two functions in a single variable θ, say $y = f(\theta)$ and $x = g(\theta)$, then the functions x and y are called *parametric functions* and θ is called the *parameter*.

3. For a function in the parametric form, say $y = f(\theta)$, $x = g(\theta)$, we have

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} \text{ and } \frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{1}{\left(\frac{dx}{d\theta}\right)} \qquad \left[\text{provided } \frac{dx}{d\theta} \neq 0\right]$$

Applications of Derivatives – I

(I) Rate of Change of Quantities

1. Average rate of change of y w.r.t. $x = \dfrac{\Delta y}{\Delta x}$.

2. Instantaneous rate of change of y w.r.t. $x = \dfrac{dy}{dx}$.

3. $\left(\dfrac{dy}{dx}\right)_{x=x_0}$ is the value of $\dfrac{dy}{dx}$ at $x = x_0$ and it represents the rate of change of y w.r.t. x at $x = x_0$.

4. **Marginal Cost (MC):** It is the rate of change of total cost (C) w.r.t. the number of units (x) produced, i.e., MC $= \dfrac{dC}{dx}$.

5. **Marginal Revenue (MR):** It is the rate of change of total revenue (R) w.r.t. the number of units (x) sold, i.e., MR $= \dfrac{dR}{dx}$.

(II) Approximations

6. $\Delta y = \left(\dfrac{dy}{dx}\right)\Delta x$ approximately.

7. Absolute error in $x = \Delta x =$ Approximate change in x.

8. Absolute error in $y = \Delta y =$ Approximate change in y.

9. Relative error in $x = \dfrac{\Delta x}{x}$.

10. Relative error in $y = \dfrac{\Delta y}{y}$.

11. Percentage error in $x = \left(\dfrac{\Delta x}{x} \times 100\right)\%$.

12. Percentage error in $y = \left(\dfrac{\Delta y}{y} \times 100\right)\%$.

Matrices

1. A *matrix* (plural *matrices*) is an ordered rectangular array (i.e., arrangement or display) of numbers or functions.
2. The elements of a matrix are always enclosed in bracket [ ] or parenthesis ( ).
3. The numbers or functions in a matrix are called *elements* or *entries* of the matrix.
4. A horizontal line of elements is called *row* of the matrix and a vertical line of elements is called *column* of the matrix.
5. A matrix having m rows and n columns is called a matrix of order $m \times n$ or simply $m \times n$ matrix (read as m by n matrix).
6. The element a_{ij} is in the i^{th} row and j^{th} column, and it is called $(i, j)^{\text{th}}$ element.
7. A matrix whose all elements are zero is called a *null matrix* or *zero matrix*. It is denoted by O.
8. A matrix is said to be a *row matrix* if it has only one row.
9. A matrix is said to be a *column matrix* if it has only one column.
10. A matrix is said to be a *square matrix* if it has same number of rows and columns.
11. The elements $a_{11}, a_{22}, \ldots, a_{nn}$ are called *diagonal elements*.
12. A square matrix $A = [a_{ij}]_{n\times n}$ is called *diagonal matrix* if all the non-diagonal elements are zero, i.e., if $a_{ij} = 0$ for $i \neq j$.
13. A diagonal matrix of order n with diagonal elements $d_1, \ldots, d_n$, is denoted by diag $[d_1, \ldots, d_n]$.
14. A square matrix $A = [a_{ij}]_{n\times n}$ is called *scalar matrix* if all the non-diagonal elements are zero and the diagonal elements are equal.
15. A square matrix $A = [a_{ij}]_{n\times n}$ is called *identity matrix* or *unit matrix* if all the non-diagonal elements are zero and diagonal elements are unity. The identity matrix of order n is denoted by I_n.
16. Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be *equal* if
    (i) A and B have same order, and
    (ii) $a_{ij} = b_{ij}$ for all i and j.
17. $A + B$ is a matrix obtained by adding the corresponding elements of matrices A and B.
18. Properties of matrix addition:
    (i) Matrix addition is commutative: If A and B are two matrices of same order, then
$$A + B = B + A.$$

(ii) Matrix addition is associative: If A, B and C are three matrices of same order, then
$$A + (B + C) = (A + B) + C.$$

(iii) Existence of Additive Identity: The null matrix O is the additive identity for matrix addition, i.e., $A + O = O + A = A$.

(iv) Existence of Additive Inverse: For every matrix $A = [a_{ij}]$, there exists a unique matrix $-A = [-a_{ij}]$ such that $A + (-A) = O = (-A) + A$. The matrix $-A$ is called the additive inverse of the matrix A.

19. $A - B$ is a matrix obtained by subtracting elements of B from the corresponding elements of A.

20. kA is a matrix obtained by multiplying each element of A by scalar k.

21. Properties of scalar multiplication:

(i) $k(A + B) = kA + kB$ (ii) $(k + l)A = kA + lA$

22. Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$ be two matrices, then we define multiplication of matrices A and B as $AB = [c_{ik}]_{m\times p}$, where c_{ik} is obtained by first taking the element-wise products of elements of i^{th} row of A and k^{th} column of B, and then adding such products.

23. The product AB is defined only if the number of columns of A and number of rows of B are same.

24. In the product AB, the matrix A is called *pre-multiplier* matrix and the matrix B is called *post-multiplier* matrix.

25. Properties of matrix multiplication:

(i) Matrix multiplication is associative: For any three matrices A, B and C, we have $(AB)C = A(BC)$ whenever both sides of above equality are defined.

(ii) Matrix multiplication is distributive over matrix addition: For any three matrices A, B and C, we have $A(B + C) = AB + AC$ and $(A + B)C = AC + BC$, whenever both sides of above equalities are defined.

(iii) Existence of multiplicative identity: For every square matrix A, there exists an identity matrix of same order such that $IA = AI = A$.

(iv) Matrix multiplication is not commutative in general: For any two matrices A and B, if both the products AB and BA are defined, it is not necessary that $AB = BA$ (i.e., Commutativity may hold in some cases and may not hold in some other cases).

(v) Zero matrix as the product of two non-zero matrices: If the product of two matrices is a zero matrix, then it is not necessary that one of the matrices is a zero matrix.

26. Transpose of matrix A is the matrix obtained by interchanging the rows and columns of A. It is also denoted by A^T or A'.

27. Properties of transpose of matrices:

(i) For any matrix A, we have $(A^T)^T = A$.

(ii) For any matrix A and scalar k, we have $(kA)^T = kA^T$.

(iii) For any two matrices A and B of same order, we have

(a) $(A + B)^T = A^T + B^T$. (b) $(A - B)^T = A^T - B^T$.

(iv) For any two matrices A and B for which AB is defined, we have $(AB)^T = B^T A^T$.

28. A square matrix A is said to be *symmetric matrix*, if $A^T = A$.

29. A square matrix A is said to be *skew symmetric matrix*, if $A^T = -A$.

30. Principle of mathematical induction: Let $P(n)$ be a statement involving natural number n such that

(i) $P(1)$ is true, and

(ii) $P(k)$ is true implies $P(k+1)$ is true.

Then, $P(n)$ is true for all natural numbers n.

31. The six elementary transformations on a matrix are:

(i) Interchange of any two rows, $R_i \leftrightarrow R_j$.

(ii) Multiplication of all elements of any row by a non-zero scalar, $R_i \to kR_i$.

(iii) Addition to the elements of any row, the corresponding elements of any other row multiplied by a non-zero scalar, $R_i \to R_i + kR_j$.

(iv) Interchange of any two columns, $C_i \leftrightarrow C_j$.

(v) Multiplication of all the elements of any column by a non-zero scalar, $C_i \to kC_i$.

(vi) Addition to the elements of any column, the corresponding elements of any other column multiplied by a non-zero scalar, $C_i \to C_i + kC_j$.

32. Let A be a square matrix of order n. If there exists a square matrix B of same order n such that $AB = BA = I_n$, then we say that A is *invertible*. The matrix B is called *inverse matrix* of A and is denoted by A^{-1}.

33. *Uniqueness of inverse*: Inverse of a square matrix, if it exists, is unique.

34. We can use either *elementary row transformations* or *elementary column transformations* to find the inverse, but both cannot be used simultaneously.

35. Let $A = PQ$, then

(i) The effect of any elementary row operation on A is same as applying this elementary row operation on P (i.e., pre-multiplier) and keeping Q unchanged.

(ii) The effect of any elementary column operation on A is same as applying this elementary column operation on Q (i.e., post-multiplier) and keeping P unchanged.

Determinants

1. Let A be any square matrix. We can associate a unique expression or number with this square matrix called *determinant* of A. It is denoted by det A or $|A|$.
2. Let $A = [a_{ij}]$ be a square matrix. The *Minor* M_{ij} of an element a_{ij} of A is the determinant of the matrix obtained by deleting i^{th} row and j^{th} column of A.
3. Minor of an element of a square matrix of order n $(n \geq 2)$ is a determinant of order $(n-1)$.
4. The *Cofactor* A_{ij} of an element a_{ij} of a square matrix $A = [a_{ij}]$ is defined as $A_{ij} = (-1)^{i+j} M_{ij}$.
5. $|A|$ = Sum of product of elements (of any row or column) with their corresponding cofactors.
6. If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero.
7. For any square matrix A of order n, we have

   (i) $|A^T| = |A|$.   (ii) $|kA| = k^n|A|$.

8. For any two square matrices A and B of same order, we have $|AB| = |A||B|$.

9. For any invertible square matrix A of order n, we have $|A^{-1}| = \dfrac{1}{|A|}$.

10. A square matrix A is said to be *singular*, if $|A| = 0$.

11. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be any three points in the XY-plane. Consider the following determinant,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}[x_1(y_2 - y_3) - y_1(x_2 - x_3) + (x_2y_3 - x_3y_2)].$$

Then,

(i) Area of triangle with vertices A, B and $C = |\Delta|$ sq. units.

(ii) Points A, B and C are said to be *collinear*, if $\Delta = 0$. (It is called *condition of collinearity* of three points.)

12. Equation of line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$

13. Let $A = [a_{ij}]$ be a square matrix of order n. Then, we define *adjoint* of A as $\text{adj}\, A = [A_{ij}]^T$, where A_{ij} denotes the cofactor of a_{ij} in A.

14. For any square matrix A of order n, we have

(i) $A(\text{adj}\, A) = (\text{adj}\, A)A = |A|I$.

(ii) $|\text{adj}\, A| = |A|^{n-1}$.

(iii) $|A(\text{adj}\, A)| = |A|^n$.

(iv) $|\text{adj}\,(\text{adj}\, A)| = |A|^{(n-1)^2}$.

(v) $\text{adj}\,(kA) = k^{n-1}(\text{adj}\, A)$, for any scalar k.

(vi) $\text{adj}\,(\text{adj}\, A) = |A|^{n-2}A$.

(vii) $\text{adj}\,(A^T) = (\text{adj}\, A)^T$.

15. For any two square matrices A and B (for which AB is defined), we have

$$\text{adj}\,(AB) = (\text{adj}\, A)(\text{adj}\, B).$$

16. A square matrix is invertible if and only if it is non-singular.

17. The inverse of a non-singular matrix A is given by $A^{-1} = \dfrac{1}{|A|}(\text{adj}\, A)$.

18. For any two square matrices A and B (for which AB is defined), we have $(AB)^{-1} = B^{-1}A^{-1}$.

19. For any square matrix A, we have

(i) $(A^T)^{-1} = (A^{-1})^T$.

(ii) $(A^{-1})^{-1} = A$.

(iii) $A^{-1}A = I = AA^{-1}$.

20. A set of values of x, y and z which simultaneously satisfy all the equations of the system is called a *solution* of the system of equations.

21. A system of equations is said to be *consistent* if it has one or more solutions. A consistent system can either have a unique solution or infinitely many solutions.

22. A system of equations is said to be *inconsistent* if it has no solution.

23. A system of equations $AX = B$ is said to be *non-homogeneous*, if $B \neq O$.

24. A system of equations $AX = B$ is said to be *homogeneous* if $B = O$.

Properties of Determinants

1. If each element in a row (or column) of a determinant is zero, then the value of the determinant is zero.
2. If any two rows (or columns) of a determinant are identical, then the value of the determinant is zero.
3. If any two rows (or columns) of a determinant are interchanged, then sign of the determinant changes.
4. If each element of a row (or column) of a determinant is multiplied by a constant k, then its value gets multiplied by k. In other words, we can take out any common factor from any row (or column) of a determinant.
5. If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.
6. The value of the determinant remains same if we apply the operations,

$$R_i \to R_i + kR_j \text{ or } C_i \to C_i + kC_j.$$